

**Aufgabe 42**

Berechne durch Substitution.

a)  $\int (3x - 1)^8 dx$

b)  $\int (2 - 5x)^{19} dx$

c)  $\int (3 - x)^5 dx$

d)  $\int (4x - 8)^{22} dx$

e)  $\int (1 - 12x)^{23} dx$

f)  $\int \frac{1}{2x - 4} dx = \int (2x - 4)^{-1} dx$

g)  $\int \frac{1}{1 - 4x} dx = \int (1 - 4x)^{-1} dx$

h)  $\int \frac{1}{3 - 5x} dx = \int (3 - 5x)^{-1} dx$

i)  $\int \frac{1}{(2x - 3)^{12}} dx = \int (2x - 3)^{-12} dx$

**Lösungen:**

**Ad a)**

**Substitution:**  $u = 3x - 1 \quad \Rightarrow \quad u' = \frac{du}{dx} = 3 \quad \Rightarrow \quad dx = -\frac{1}{3}du$

**Einsetzen:**  $\int (3x - 1)^8 dx = \int u^8 \cdot \frac{1}{3} du = \frac{1}{3} \int u^8 du = \frac{u^9}{27} + c = -\frac{(3x - 1)^9}{27} + c$

**Ad b)**

**Substitution:**  $u = 2 - 5x \quad \Rightarrow \quad u' = \frac{du}{dx} = -5 \quad \Rightarrow \quad dx = -\frac{1}{5}du$

**Einsetzen:**  $\int (2 - 5x)^{19} dx = -\int u^{19} \cdot \frac{1}{5} du = -\frac{1}{5} \int u^{19} du = -\frac{u^{20}}{100} + c = -\frac{(2 - 5x)^{20}}{100} + c$

**Ad c)**

**Substitution:**  $u = 3 - x \quad \Rightarrow \quad u' = \frac{du}{dx} = -1 \quad \Rightarrow \quad dx = -du$

**Einsetzen:**  $\int (3 - x)^5 dx = -\int u^5 du = -\int u^5 du = -\frac{u^6}{6} + c = -\frac{(3 - x)^6}{6} + c$

**Ad d)**

**Substitution:**  $u = 4x - 8 \quad \Rightarrow \quad u' = \frac{du}{dx} = 4 \quad \Rightarrow \quad dx = \frac{1}{4}du$

**Einsetzen:**  $\int (4 - x)^{22} dx = \int u^{22} \cdot \frac{1}{4} du = \frac{1}{4} \int u^{22} du = \frac{u^{23}}{92} + c = \frac{(4x - 8)^{23}}{92} + c$

**Ad e)**

**Substitution:**  $u = 1 - 12x \quad \Rightarrow \quad u' = \frac{du}{dx} = -12 \quad \Rightarrow \quad dx = -\frac{1}{12}du$

**Einsetzen:**

$$\int (1 - 12x)^{23} dx = - \int u^{23} \cdot \frac{1}{12} du = -\frac{1}{12} \int u^{23} du = -\frac{u^{24}}{288} + c = -\frac{(1 - 12x)^{24}}{288} + c$$

**Ad f)**

**Substitution:**  $u = 2x - 4 \quad \Rightarrow \quad u' = \frac{du}{dx} = 2 \quad \Rightarrow \quad dx = \frac{1}{2}du$

**Einsetzen:**

$$\begin{aligned} \int (2x - 4)^{-1} dx &= \int u^{-1} \cdot \frac{1}{2} du &= \\ &= \frac{1}{2} \int u^{-1} du &= \\ &= \frac{1}{2} \cdot \ln|u| + c_1 &= \\ &= \frac{1}{2} \cdot \ln|2x - 4| + c_1 &= \\ &= \frac{1}{2} \cdot \ln(2 \cdot |x - 2|) + c_1 &= \\ &= \frac{1}{2} \cdot \ln|x - 2| + \underbrace{\ln 2 + c_1}_{=c} &= \\ &= \frac{1}{2} \cdot \ln|x - 2| + c &= \end{aligned}$$

**Ad g)**

**Substitution:**  $u = 1 - 4x \quad \Rightarrow \quad u' = \frac{du}{dx} = -4 \quad \Rightarrow \quad dx = -\frac{1}{4}du$

**Einsetzen:**

$$\begin{aligned}
 \int (1 - 4x)^{-1} dx &= - \int u^{-1} \cdot \frac{1}{4} du &= \\
 &= -\frac{1}{4} \int u^{-1} du &= \\
 &= -\frac{1}{4} \cdot \ln|u| + c_1 &= \\
 &= -\frac{1}{4} \cdot \ln|1 - 4x| + c_1
 \end{aligned}$$

**Ad h)**

**Substitution:**  $u = 3 - 5x \Rightarrow u' = \frac{du}{dx} = -5 \Rightarrow dx = -\frac{1}{5}du$

**Einsetzen:**

$$\begin{aligned}
 \int (3 - 5x)^{-1} dx &= - \int u^{-1} \cdot \frac{1}{4} du &= \\
 &= -\frac{1}{5} \int u^{-1} du &= \\
 &= -\frac{1}{5} \cdot \ln|u| + c_1 &= \\
 &= -\frac{1}{5} \cdot \ln|3 - 5x| + c_1
 \end{aligned}$$

**Ad i)**

**Substitution:**  $u = 2x - 3 \Rightarrow u' = \frac{du}{dx} = 2 \Rightarrow dx = \frac{1}{2}du$

**Einsetzen:**

$$\begin{aligned}
 \int (2x - 3)^{-12} dx &= \int u^{-12} \cdot \frac{1}{4} du &= \\
 &= \frac{1}{2} \int u^{-12} du &= \\
 &= -\frac{1}{22} \cdot u^{-11} + c_1 &= \\
 &= -\frac{1}{22 \cdot (2x - 3)^{11}} + c_1
 \end{aligned}$$