

Aufgabe 44

Berechne das unbestimmte Integral.

a) $\int 5x \cdot \ln(2x) dx$

b) $\int 3x \cdot \ln(4x) dx$

c) $\int x \cdot \sin(2x) dx$

d) $\int x \cdot \cos(2x) dx$

e) $\int 3x \cdot \sin(4x) dx$

f) $\int 6x \cdot \cos(2x) dx$

Lösungen:

Ad a)

$$\left. \begin{array}{l} f(x) = 5x \Rightarrow F(x) = \frac{5}{2}x^2 \\ g(x) = \ln(2x) \Rightarrow g'(x) = \frac{1}{x} \end{array} \right\} \Rightarrow$$

$$\begin{aligned} \int 5x \cdot \ln(2x) dx &= F(x) \cdot g(x) - \int F(x) g'(x) dx = \\ &= \frac{5}{2} \cdot x^2 \cdot \ln(2x) - \int \frac{5}{2} \cdot x^2 \cdot \frac{1}{x} dx = \\ &= \frac{5}{2} \cdot x^2 \cdot \ln(2x) - \frac{5}{2} \cdot \int x dx = \\ &= \frac{5}{2} \cdot x^2 \cdot \ln(2x) - \frac{5}{2} \cdot \frac{x^2}{2} + c = \\ &= \frac{5}{2} \cdot x^2 \cdot \left(\ln(2x) - \frac{1}{2} \right) + c \end{aligned}$$

Ad b)

$$\left. \begin{array}{l} f(x) = 3x \Rightarrow F(x) = \frac{3}{2}x^2 \\ g(x) = \ln(4x) \Rightarrow g'(x) = \frac{1}{x} \end{array} \right\} \Rightarrow$$

$$\begin{aligned} \int 3x \cdot \ln(4x) dx &= F(x) \cdot g(x) - \int F(x) g'(x) dx = \\ &= \frac{3}{2}x^2 \cdot \ln(4x) - \int \frac{3}{2}x^2 \cdot \frac{1}{x} dx = \\ &= \frac{3}{2}x^2 \cdot \ln(4x) - \int \frac{3x}{2} dx = \\ &= \frac{3}{2}x^2 \cdot \ln(4x) - \frac{3x^2}{4} + c = \\ &= \frac{3}{2}x^2 \cdot \left(\ln(4x) - \frac{1}{2} \right) + c \end{aligned}$$

Ad c)

$$\left. \begin{array}{lcl} f(x) & = & \sin(2x) \Rightarrow F(x) & = & -\frac{1}{2}\cos(2x) \\ g(x) & = & x & \Rightarrow & g'(x) & = & 1 \end{array} \right\} \Rightarrow$$

$$\begin{aligned} \int x \cdot \sin(2x) dx &= F(x) \cdot g(x) - \int F(x) g'(x) dx = \\ &= \frac{1}{2}\cos(2x) \cdot x + \int \frac{1}{2}\cos(2x) \cdot 1 dx = \\ &= -\frac{x}{2}\cos(2x) + \frac{1}{4}\sin(2x) = \\ &= -\frac{1}{2}x\cos(2x) + \frac{1}{4}\sin(2x) + c \end{aligned}$$

Ad d)

$$\left. \begin{array}{lcl} f(x) & = & \cos(2x) \Rightarrow F(x) & = & \frac{1}{2}\sin(2x) \\ g(x) & = & x & \Rightarrow & g'(x) & = & 1 \end{array} \right\} \Rightarrow$$

$$\begin{aligned} \int x \cdot \cos(2x) dx &= F(x) \cdot g(x) - \int F(x) g'(x) dx = \\ &= \frac{1}{2}\sin(2x) \cdot x + \int \frac{1}{2}\sin(2x) \cdot 1 dx = \\ &= \frac{1}{2}x \cdot \sin(2x) + \frac{1}{4}\cos(2x) + c \end{aligned}$$

Ad e)

$$\left. \begin{array}{lcl} f(x) & = & \sin(4x) \Rightarrow F(x) & = & -\frac{1}{4}\cos(4x) \\ g(x) & = & 3x & \Rightarrow & g'(x) & = & 3 \end{array} \right\} \Rightarrow$$

$$\begin{aligned} \int 3x \cdot \sin(4x) dx &= F(x) \cdot g(x) - \int F(x) g'(x) dx = \\ &= -\frac{1}{4}\cos(4x) \cdot 3x + \int \frac{1}{4}\cos(4x) \cdot 3 dx = \\ &= -\frac{3}{4}x \cdot \sin(4x) + \frac{3}{16}\cos(4x) + c \end{aligned}$$

Ad f)

$$\left. \begin{array}{l} f(x) = \cos(2x) \Rightarrow F(x) = \frac{1}{2}\sin(2x) \\ g(x) = 6x \Rightarrow g'(x) = 6 \end{array} \right\} \Rightarrow$$
$$\begin{aligned} \int 6x \cdot \cos(2x) dx &= F(x) \cdot g(x) - \int F(x) g'(x) dx = \\ &= 3\sin(2x) \cdot x - \int 3\sin(2x) dx = \\ &= 3x\sin(2x) + \frac{3}{2}\cos(2x) + c \end{aligned}$$