

Aufgabe 93

Beweise die angegebene Aussage.

a) $\int_a^b f(x) dx - \int_b^a f(x) dx = 2 \cdot \int_a^b f(x) dx$

b) $\int_0^4 f(x) dx + \int_2^4 f(x) dx + \int_2^4 f(x) dx = 3 \cdot \int_2^4 f(x) dx + \int_0^2 f(x) dx$

c) $\int_0^4 2 \cdot f(x) dx + \int_0^4 3 \cdot f(x) dx - \int_4^0 4 \cdot f(x) dx = 9 \cdot \int_0^4 f(x) dx$

Lösungen:

Ad a)

$$\begin{aligned}
 \int_a^b f(x) dx - \int_b^a f(x) dx &= F(b) - F(a) - (F(a) - F(b)) = \\
 &= F(b) - F(a) - F(a) + F(b) = \\
 &= 2 \cdot F(b) - 2 \cdot F(a) = \\
 &= 2 \cdot (F(b) - F(a)) = \\
 &= 2 \cdot \int_a^b f(x) dx
 \end{aligned}$$

Ad b)

$$\begin{aligned}
 \int_0^4 f(x) dx + \int_2^4 f(x) dx + \int_2^4 f(x) dx &= \int_0^4 f(x) dx + 2 \cdot \int_2^4 f(x) dx = \\
 &= \int_0^2 f(x) dx + \int_2^4 f(x) dx + 2 \cdot \int_2^4 f(x) dx = \\
 &= \int_0^2 f(x) dx + 3 \cdot \int_2^4 f(x) dx
 \end{aligned}$$

Ad c)

$$\begin{aligned}
 \int_0^4 2f(x) dx + \int_0^4 3f(x) dx - \int_{-4}^0 4f(x) dx &= 2 \int_0^4 f(x) dx + 3 \int_0^4 f(x) dx - 4 \int_{-4}^0 f(x) dx = \\
 &= 5 \int_0^4 f(x) dx + 4 \cdot \int_0^4 f(x) dx = \\
 &= 9 \int_0^4 f(x) dx
 \end{aligned}$$